Barem clasa a VIII a

1) a) $1+\frac{1}{k^2}+\frac{1}{(k+1)^2}=\frac{k^4+3k^2+2k^3+2k+1}{k^2(k+1)^2}=\frac{(k^2+k+1)^2}{k^2(k+1)^2}$ (2p)

$\sqrt{\frac{(k^2+k+1)^2}{k^2(k+1)^2}}=\frac{k^2+k+1}{k(k+1)}=\frac{k(k+1)+1}{k(k+1)}=1+\frac{1}{k(k+1)}$ 1p

$b) S=\frac{3}{2}+\sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}}+\sqrt{1+\frac{1}{3^2}+\frac{1}{4^2}}+\ldots\ldots\ldots+\sqrt{1+\frac{1}{99^2}+\frac{1}{100^2}}$

$\frac{3}{2}+\frac{2^2+2+1}{2\cdot3}+\frac{3^2+3+1}{3\cdot4}+\ldots\ldots\ldots\ldots\ldots+\frac{99^2+99+1}{99(99+1)}$ (1P)

$S=(1+\frac{1}{1.2})+(1+\frac{1}{2.3})+\ldots\ldots\ldots(1+\frac{1}{99.100})$(1p)

$S=99+1-\frac{1}{100}$ (1p)

$S=\frac{99.101}{100}$ (1p)

2) a) $\sqrt{a}+\sqrt{b}=c|\ (\sqrt{a}-\sqrt{b})$ .....(1p)

$a-b=c(\sqrt{a}-\sqrt{b}) \Rightarrow \sqrt{a}-\sqrt{b}=\frac{a-b}{c}$

$\sqrt{a}+\sqrt{b}=c$

$\sqrt{a}-\sqrt{b}=\frac{a-b}{c}$

$2\sqrt{a}=\frac{a-b+c^2}{c} \Rightarrow \sqrt{a}=\frac{a-b+c^2}{2c}$ .....(1p)

$\sqrt{b}=\frac{b-a+c^2}{2c}$ ..... (1p)

$b)$ daca n=0 $\Rightarrow A=11-3^{2m} \in \mathbb{N}$ daca $m \in \{0;1\}$ .....(1p)

Pt m=0⇒ A=10=3+7

m=1 A=2- nu poate fi scris ca suma de doua numere prime

pt. n>0 ⇒A este impar ⇒ A = 2+p cu p nr. prim ⇒

$\Rightarrow p=2^n-3^{2n}+2^{n+2}+4=(2^n+2)^2-3^{2m}=(2^n+3^m+2)(2^n-3^m+2)$

$2^n+3^m+2>1 \Rightarrow 2^n-3^m+2=1 \Rightarrow 2^n+3^m=p$..... (1p)

$2^n+1=3^m \Rightarrow$ n este impar

Pt n=1 se obţine m=1..... (1p)

Pt $n \geq 3 \Rightarrow 2^n+1=\mathcal{M}_4+1 \Rightarrow 3^m \in \mathcal{M}_4+(-1)^n \Rightarrow$ m este par⇒m=2k

$2^n+1=3^{2k} \Rightarrow 2^n=3^k-1=(3^k-1)(3^k+1) \Rightarrow n=3$ si k=1⇒m=2⇒p=19

$(n;m) \in \{(0,0);(1,1);(3,2)\}$ ..... (1p)

3) ABCD dreptunghi⇒AC=10, deci ΔMAC isoscel (2p)
(AG bisectoare, deci (AG) este mediană.
FG- linie mijlocie în ΔMBC, deci FG||BC, FG=$\frac{BC}{2}$ (1p)
FG||(BCD) (1p)
EF||(BCD) (1p)
ΔEFG~ΔBCD⇒ $\frac{A\Delta EFG}{A\Delta BCD} = (\frac{FG}{BC})^2 = \frac{1}{4}$ (2p)

4)a) $\frac{ED}{EB}=\frac{3}{5}$ DC+BC=16
th.bis
$\Delta BCD \Rightarrow \frac{DC}{BC} = \frac{DE}{BC}$ ⇒ DC=6
BC=10

AD=5
AB=13 (1p)
T.P
$\Delta ABD \Rightarrow$ DB=12
$A_{\Delta ABD}$=30 (1p)
$DF = \frac{60}{13}$
Finalizare $A_{ABCD}=\frac{570}{13}$ (1p)

b) VE=$\frac{BC+CD}{2}$=8 cm (1p)
T3⊥
EP ⊥ AB ⇒ VP⊥AB (1p)
⇒ $\Delta BEP \sim \Delta BDF$ (1p)
EP=$\frac{75}{26}$
Finalizare VP=$\frac{\sqrt{48889}}{26}$ (1p)